basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

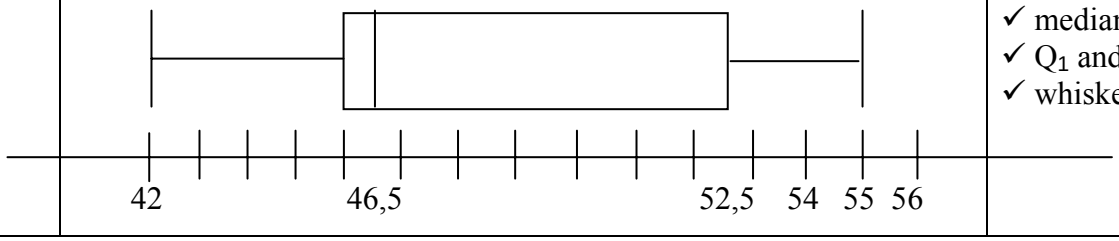
FEBRUARY/MARCH 2011

MEMORANDUM

MARKS: 150

This memorandum consists of 15 pages.

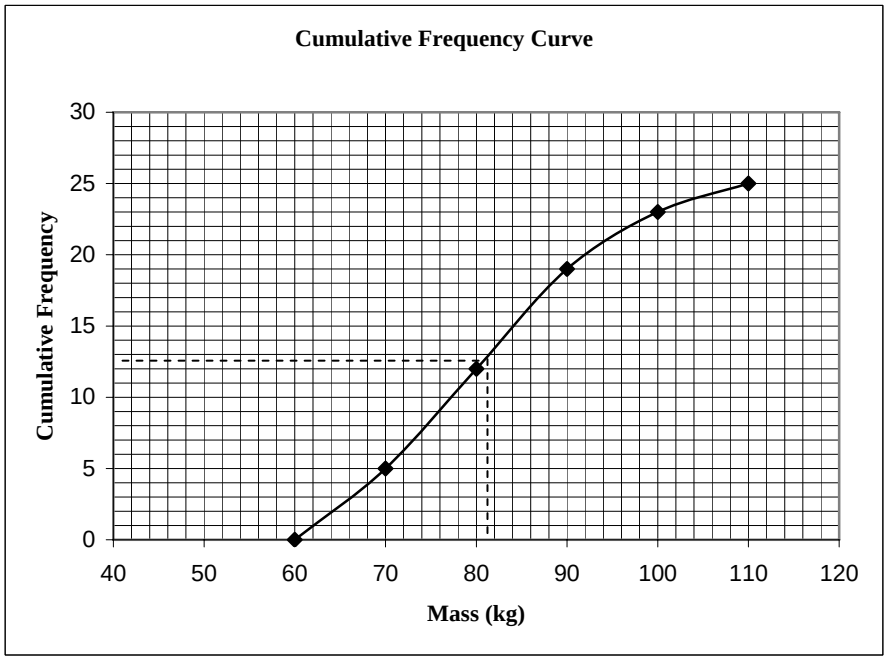
QUESTION 1

1.1	$\frac{55+55+50+47+42+3x}{8} = 48{,}375$ $\frac{249+3x}{8} = 48{,}375$ $3x = 138$ $x = 46$	✓ $\frac{249+3x}{8} = 48{,}375$ ✓ $3x = 138$ (2)
1.2		✓ max and min ✓ median ✓ Q_1 and Q_3 ✓ whiskers (4) **[6]**

QUESTION 2

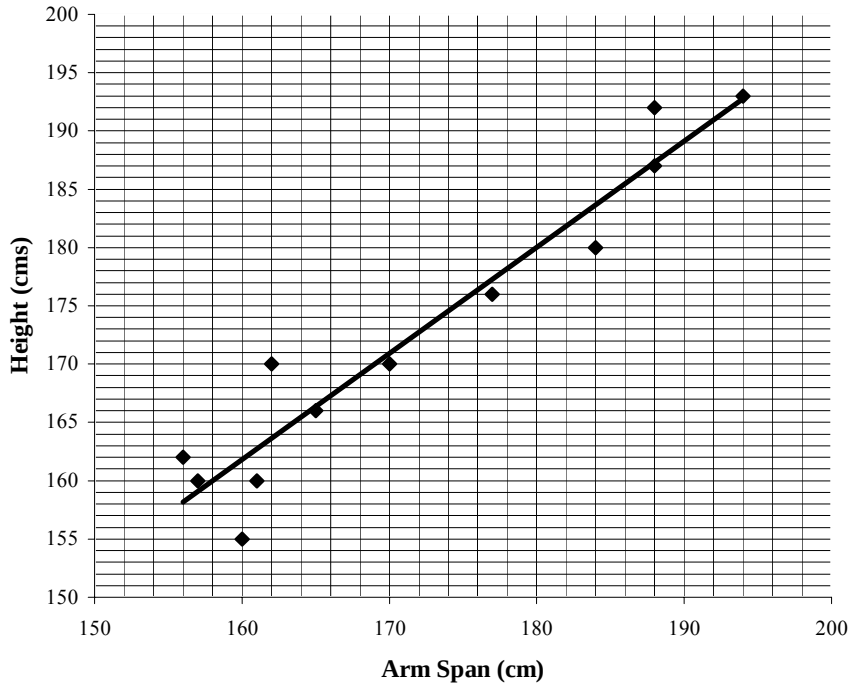
2.1		✓✓ Frequencies ✓✓ Cumulative Frequencies (4)

Mass (kg)	Frequency	Cumulative Frequency
$60 \le x < 70$	5	5
$70 \le x < 80$	7	12
$80 \le x < 90$	7	19
$90 \le x < 100$	4	23
$100 \le x < 110$	2	25

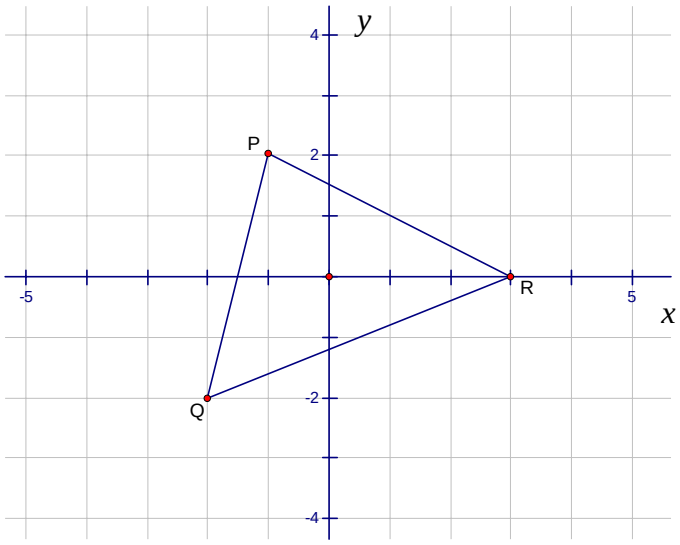
2.2		✓✓ plotting points 1 mark: 3 – 5 points correctly 0 marks : 2 or less points correctly plotted ✓ graph (3)
2.3	Mean = 79,28	✓✓ answer (2)

2.4	Standard Deviation = 11,02 79,28 – 11,02 = 68,26 79,28+11,02 = 90,3 17 players lie in this interval. $\frac{17}{25} = 68\%$	✓✓✓ sd = 11,02 ✓ 17 players ✓ 68% (5) **[14]**

QUESTION 3

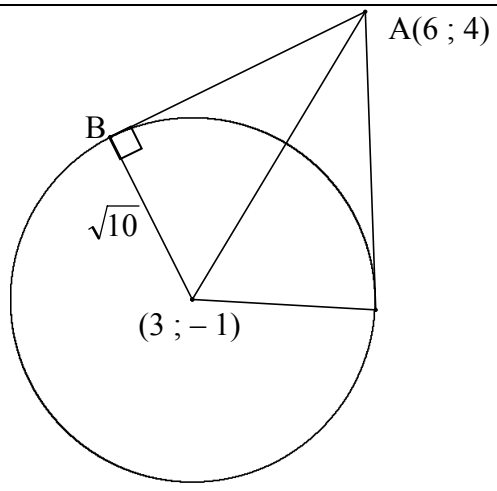
3.1 & 3.2		Question 3.1 4 marks: All points plotted correctly. 3 marks: 9 – 11 points correct 2 marks: 6 – 8 points correct 1 marks: 3 – 5 points correct 0 marks if less than 3 points plotted correctly. (4) Question 3.2 ✓✓ linear best fit Line (2)
3.3	Yes. The relationship between arm span and height is a positive, linear one so we can expect a person with below average arm span to have below average in height.	✓ Yes ✓ Reason (2) **[8]**

QUESTION 4



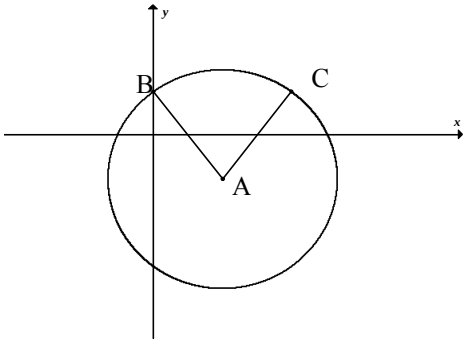
4.1	Let β be the angle of inclination of PQ. $\tan\beta = m_{PQ}$ $\tan\beta = \frac{2-(-2)}{-1-(-2)}$ $\tan\beta = 4$ $\beta = 75{,}96°$	✓ $\tan\beta = m_{PQ}$ ✓ $\tan\beta = 4$ ✓ answer (3)
4.2	$M\left(\frac{-1+3}{2};\frac{2+0}{2}\right)$ M (1 ; 1)	✓ *x*-value ✓ *y*-value (2)
4.3	$PQ = \sqrt{(-1+2)^2+(2+2)^2}$ $= \sqrt{17}$ $PR = \sqrt{(-1-3)^2+(2-0)^2}$ $= \sqrt{20}$ $QR = \sqrt{(0-(-2))^2+(3-(-2))^2}$ $= \sqrt{29}$ Perimeter = $\sqrt{29}+\sqrt{20}+\sqrt{17}$ = 13,98 units = 14 to the nearest whole number	✓ substitution into correct formula ✓ answer ✓ answer ✓ sum ✓ answer (5)
4.4	$y-1 = 4(x-1)$ $y = 4x-3$	✓ $m = 4$ ✓ substitution of (1 ; 1) ✓ answer (3) **[13]**

QUESTION 5

5.1.1	$x^2 + y^2 - 8x + 6y$ $= (2)^2 + (-9)^2 - 8(2) + 6(-9)$ $= 4 + 81 - 16 - 54$ $= 15$ Hence, the point lies on the circumference of the circle. **OR** $x^2 + y^2 - 8x + 6y = 15$ $(x-4)^2 + (y+3)^2 = 15 + 16 + 9$ $(x-4)^2 + (y+3)^2 = 40$ $(x-4)^2 + (y+3)^2$ $= (2-4)^2 + (-9+3)^2$ $= 2^2 + 6^2$ $= 40$ $\therefore$ The point lies on the circumference of the circle.	✓ substitution ✓ answer (2) ✓ substitution ✓ answer (2)
5.1.2	$x^2 + y^2 - 8x + 6y = 15$ $(x-4)^2 + (y+3)^2 = 15 + 16 + 9$ $(x-4)^2 + (y+3)^2 = 40$ Circle centre $(4\ ;-3)$ $m_{rad} = \frac{-3-(-9)}{4-2}$ $m_{rad} = 3$ $m_{\tan} = -\frac{1}{3}$ $y + 9 = -\frac{1}{3}(x-2)$ $y = -\frac{1}{3}x - \frac{25}{3}$	✓✓ $(x-4)^2 + (y+3)^2 = 40$ ✓ centre ✓ gradient of radius ✓ gradient of tangent ✓substitution ✓ answer (7)
5.2		✓ radius = $\sqrt{10}$

	Radius AB $=\sqrt{10}$ Distance from A to centre of circle is $=\sqrt{(6-3)^2+(4+1)^2}$ $=\sqrt{9+25}$ $=\sqrt{34}$ $AB^2 = 34-10$ $AB^2 = 24$ $AB = \sqrt{24}$ $AB = 2\sqrt{6}$ $AB = 4{,}90$ **OR** $r^2 = 10$ $r = \sqrt{10}$ Radius $\perp$ tangent By Pythagoras $AB^2 = (6-3)^2 + (4+1)^2 - 10$ $= 24$ $AB = 4{,}90$	✓ subs into distance formula ✓ $\sqrt{34}$ ✓ $AB^2 = 34-10$ ✓ answer (5) ✓ $r = \sqrt{10}$ ✓✓ $AB^2 = (6-3)^2 + (4+1)^2 - 10$ ✓ $AB = 4{,}90$ (5) **[14]**

QUESTION 6

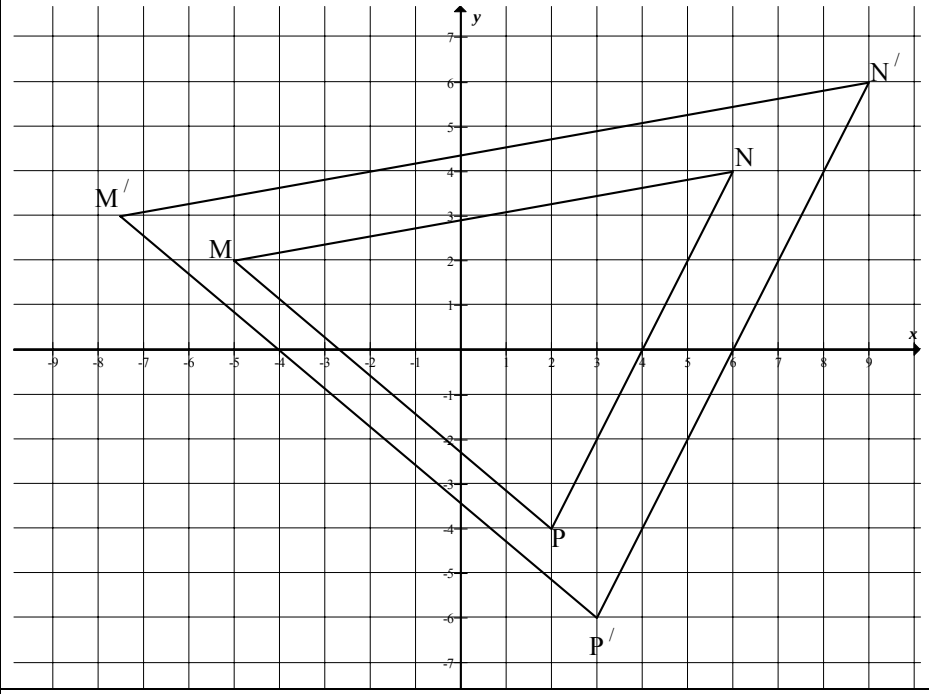


6.1	$9+(y+2)^2=25$ $(y+2)^2=16$ $y+2=\pm 4$ $y=2$ or $y=-6$ B(0 ; 2) **OR** $x=0$ $(0)^2-6(0)+y^2+4y=12$ $y^2+4y-12=0$ $(y+6)(y-2)=0$ $y=-6$ or $y=2$ B(0 ; 2)	✓ $x=0$ ✓ factors ✓ answers ✓ answer for B (4) ✓ $x=0$ ✓ factors ✓ answers ✓ answer for B (4)
6.2	C(6 ; 2)	✓✓ answer (2)
6.3	$\left(x-3\times\frac{3}{2}\right)^2+\left(y+2\times\frac{3}{2}\right)^2=\left(5\times\frac{3}{2}\right)^2$ $\left(x-\frac{9}{2}\right)^2+(y+3)^2=\left(\frac{15}{2}\right)^2$ $\left(x-\frac{9}{2}\right)^2+(y+3)^2=56{,}25$	✓ each part $\times\frac{3}{2}$ ✓ answer (2)
6.4.1	$AB=\sqrt{(12-3)^2+(10-(-2))^2}$ $=\sqrt{9^2+12^2}$ $=15$	✓ substitution ✓ answer (2)
6.4.2	The radii are 5 and 10. $r_A+r_B=5+10$ $=15$ $=$ AB The circles will only intersect at one point.	✓ addition of radii ✓ answer (2) **[12]**

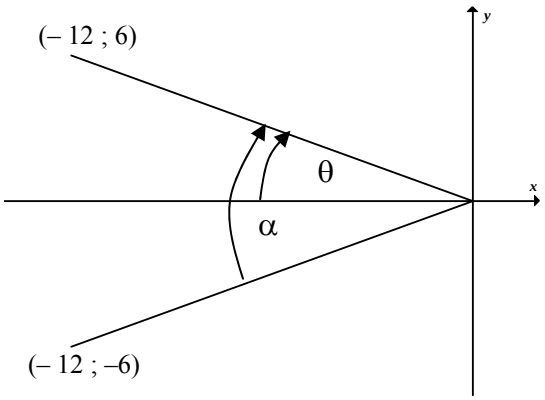
QUESTION 7

	$-3 = x\cos 150° - 2\sin 150°$	✓ expansion
	$-3 = -x.\frac{\sqrt{3}}{2} - 2.\frac{1}{2}$	✓ substitution
	$\frac{\sqrt{3}}{2}x = 2$	
	$x = \frac{4}{\sqrt{3}}$	✓ answer
	$y = x.\sin 150° + 2.\cos 150°$	✓ expansion
	$y = \frac{4}{\sqrt{3}}.\frac{1}{2} + 2.\left(-\frac{\sqrt{3}}{2}\right)$	
	$= \frac{2}{3}.\sqrt{3} - \sqrt{3}$	
	$= -\frac{\sqrt{3}}{3}$	✓ answer
		[5]

QUESTION 8

8.1		✓✓✓ coordinates of new points (3)
8.2.1	$\frac{MN}{M'N'} = \frac{2}{3}$	✓✓ (2)
8.2.2	$\frac{\text{area } \Delta MNP}{\text{area } \Delta M'N'P'} = \frac{4}{9}$	✓✓ (2)
8.2.3	$\frac{\text{area } \Delta MNP}{\text{area } \Delta M''N''P''} = \left(\frac{4}{9}\right)^{n+1}$	✓✓ (2) **[9]**

QUESTION 9

9.1	$A^{/}(-12;-6)$	✓ answer (1)
9.2	$x^{/} = x\cos\alpha - y\sin\alpha$ $-12\cos\alpha - 6\sin\alpha = -12$ $-2\cos\alpha - \sin\alpha = -2$.....(1) $y^{/} = y\cos\alpha + x\sin\alpha$ $6\cos\alpha - 12\sin\alpha = -6$ $\cos\alpha = 2\sin\alpha - 1$ … (2) Substitute (2) into (1) $-2(2\sin\alpha - 1) - \sin\alpha = -2$ $-4\sin\alpha + 2 - \sin\alpha = -2$ $-5\sin\alpha = -4$ $\sin\alpha = \frac{4}{5}$ $\alpha = 53{,}13°$ **OR**  $\tan\theta = \frac{1}{2}$ $\theta = 26{,}565°$ $\alpha = 2(26{,}565°)$ $\alpha = 53{,}13°$	✓ substitution ✓ substitution ✓ simplification ✓ substitution ✓ simplification ✓ answer (6) ✓✓ $\tan\theta = \frac{1}{2}$ ✓ $\theta = 26{,}565°$ ✓✓ $\alpha = 2(26{,}565°)$ ✓ answer (6) **[7]**

QUESTION 10

10.1.1	$\cos 28° = \sqrt{1-\sin^2 28°}$ $= \sqrt{1-a^2}$	✓ $\sqrt{1-\sin^2 28°}$ ✓answer (2)
10.1.2	$\cos 64°$ $= \cos 2(32°)$ $= 2\cos^2 32° - 1$ $= 2b^2 - 1$	✓ $\cos 2(32°)$ ✓ $2\cos^2 32° - 1$ ✓answer (3)
10.1.3	$\sin 4°$ $= \sin(32° - 28°)$ $= \sin 32°\cos 28° - \cos 32°\sin 28°$ $= \sqrt{1-b^2}.\sqrt{1-a^2} - ab$ **OR** $\sin 4°$ $= \sin(60° - 2\times 28°)$ $= \sin 60°\cos(2\times 28°) - \cos 60°\sin(2\times 28°)$ $= \frac{\sqrt{3}}{2}(1-2a^2) - \frac{1}{2}(2a)\sqrt{1-a^2}$ $= \frac{\sqrt{3}}{2} - \sqrt{3}a^2 - a\sqrt{1-a^2}$ **OR** $\sin 4°$ $= \sin(2\times 32° - 60°)$ $= \sin(2\times 32°)\cos 60° - \cos(2\times 32°).\sin 60°$ $= 2.b\sqrt{1-b^2}.\frac{1}{2} - \frac{\sqrt{3}}{2}(2b^2-1)$ $= b\sqrt{1-b^2} - \sqrt{3}b^2 + \frac{\sqrt{3}}{2}$ **OR** Using sin(A+B) + sin(A – B) = 2.sinA.cosB With A = 28° and B = 32° $\sin 60° + \sin(-4°) = 2ab$ $\sin 4° = \frac{\sqrt{3}}{2} - 2ab$ **OR**	✓ $\sin(32° - 28°)$ ✓ expansion ✓✓answer (4)

	Using sin(A+B) + sin(A – B) = 2.sinA.cosB With A = 32° and B = 28° $\sin 60° + \sin(4°) = 2\sqrt{1-b^2}.\sqrt{1-a^2}$ $\sin 4° = 2\sqrt{1-b^2}.\sqrt{1-a^2} - \frac{\sqrt{3}}{2}$ **OR** Using $\sin 4° = 2\sin 2°.\cos 2°$ and $\sin 2° = \sin(30° - 28°) = \frac{1}{2}\left(\sqrt{1-a^2} - \sqrt{3}a\right)$ and $\sin 2° = \sin(32° - 30°) = \frac{1}{2}\left(\sqrt{3}\sqrt{1-b^2} - b\right)$ and $\cos 2° = \cos(30° - 28°) = \frac{1}{2}\left(\sqrt{3}\sqrt{1-a^2} + a\right)$ and $\cos 2° = \cos(32° - 30°) = \frac{1}{2}\left(\sqrt{3}b + \sqrt{1-b^2}\right)$ then $\sin 4° = \frac{1}{2}\left\{\sqrt{3}b\sqrt{1-a^2} - 3ab + \sqrt{1-a^2}.\sqrt{1-b^2} - \sqrt{3}a\sqrt{1-b^2}\right\}$ **OR** $\sin 4° = \frac{1}{2}\left\{3\sqrt{1-b^2}\sqrt{1-a^2} + \sqrt{3}a\sqrt{1-b^2} - \sqrt{3}b\sqrt{1-a^2} - ab\right\}$	
10.2	$b\sqrt{1-a^2} - a\sqrt{1-b^2}$ $= \cos 32°.\sqrt{1-\sin^2 28°} - \sin 28°\sqrt{1-\cos^2 32°}$ $= \cos 32°.\cos 28° - \sin 28°.\sin 32°$ $= \cos(32° + 28°)$ $= \cos 60°$ $= \frac{1}{2}$	✓ substitution ✓ cos 28° ✓ sin 32° ✓ compound angle formula (4)
10.3.1	$\frac{\sin 130°.\tan 60°}{\cos 540°.\tan 230°.\sin 400°}$ $= \frac{\sin 50° \times \tan 60°}{\cos 180° \times \tan 50° \times \sin 40°}$ $= \frac{\sin 50° \times \sqrt{3}}{-1 \times \frac{\sin 50°}{\cos 50°} \times \cos 50°}$ $= -\frac{\sqrt{3}\cos 50°}{\cos 50°}$ $= -\sqrt{3}$	✓ sin 50° ✓ tan 50° ✓ sin40° ✓ cos50° ✓ $\frac{\sin 50°}{\cos 50°}$ ✓ -1 ✓ answer (7)

10.3.2	$(1-\sqrt{2}\sin 75°)(1+\sqrt{2}\sin 75°)$ $=1-2\sin^2 75°$ $=\cos 150°$ $=\frac{-\sqrt{3}}{2}$ **OR** $\sin 75°$ $=\sin(45°+30°)$ $=\sin 45°.\cos 30°+\cos 45°.\sin 30°$ $=\frac{\sqrt{2}}{2}.\frac{\sqrt{3}}{2}+\frac{\sqrt{2}}{2}.\frac{1}{2}$ $\sqrt{2}\sin 75°=\frac{\sqrt{3}}{2}+\frac{1}{2}=a$ $(1-\sqrt{2}\sin 75°)(1+\sqrt{2}\sin 75°)$ $=(1-a)(1+a)$ $=1-a^2$ $=1-\left(\frac{3}{4}+\frac{1}{4}+2.\frac{\sqrt{3}}{2}.\frac{1}{2}\right)$ $=-\frac{\sqrt{3}}{2}$	✓simplification ✓$1-2\sin^2 75°$ ✓$\cos 150°$ ✓answer (4) ✓simplification ✓$1-2\sin^2 75°$ ✓$\cos 150°$ ✓answer (4)
10.4	$\sin^2 x+\cos 2x-\cos x=0$ $\sin^2 x+(\cos^2 x-\sin^2 x)-\cos x=0$ $\cos^2 x-\cos x=0$ $\cos x(\cos x-1)=0$ $\cos x=0$ *or* $\cos x=1$ $x=\pm 90°+k.360°$ *or* $x=0°+k.360°$ $k\in Z$ $=k.360°$ (i.e. $x=90°+k.180°$ or $x=k.360°\pm 90°, k\in Z$)	✓$(\cos^2 x-\sin^2 x)$ ✓$\cos^2 x-\cos x=0$ ✓factors ✓$\cos x=0$ *or* $\cos x=1$ ✓$90°+k.360°$ ✓$k.360°$ ✓ $x=-90°+k.360°$ (7)
10.5.1	$x=0°;90°;180°$	✓✓✓ each value (3)

10.5.2	$\frac{\cos 2x.\tan x}{\sin^2 x} = \frac{(\cos^2 x - \sin^2 x).\frac{\sin x}{\cos x}}{\sin^2 x}$ $= \frac{\cos^2 x - \sin^2 x}{\cos x.\sin x}$ $= \frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}$ $= \frac{\cos x}{\sin x} - \tan x$	✓ $(\cos^2 x - \sin^2 x)$ ✓ $\frac{\sin x}{\cos x}$ ✓ answer ✓ $\frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}$ ✓answer (5) **[39]**

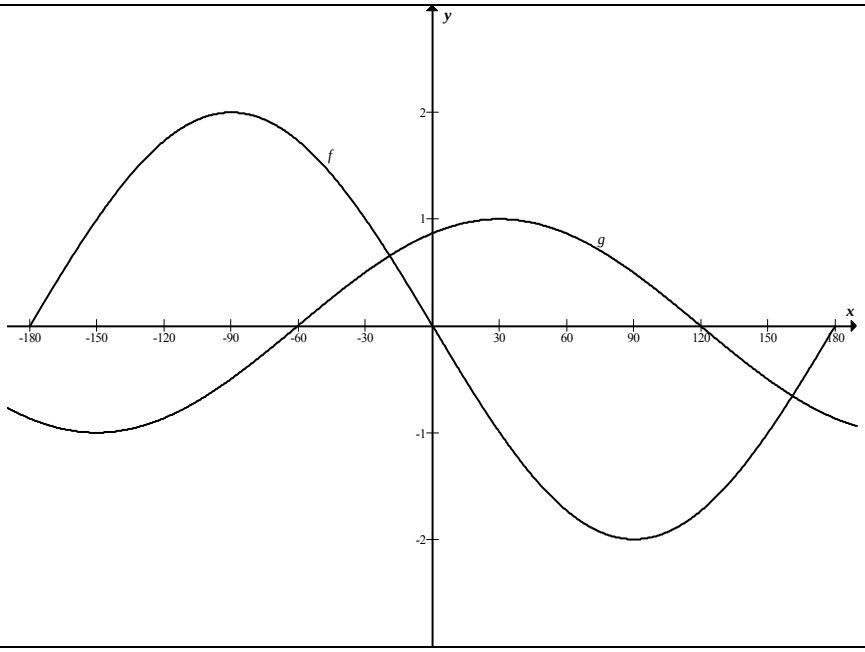
QUESTION 11

11.1	$EC^2 = DE^2 + DC^2 - 2DE.DC\cos\hat{C}$ $= (7,5)^2 + (9,4)^2 - 2.(7,5)(9,4)\cos 32°$ $= 25,03521844...$ $EC = 5,0$ metres	✓substitution into cosine rule ✓ 25,03521844... ✓answer (3)
11.2	$\frac{\sin D\hat{C}E}{7,5} = \frac{\sin 32°}{5,0}$ $\sin D\hat{C}E = \frac{7,5.\sin 32°}{5,0}$ $= 0,7948788963$ $D\hat{C}E = 52,6°$	✓ sin rule ✓ 0,7948788963 ✓answer (3)
11.3	Area of ΔDEC $= \frac{1}{2}DE.DC\sin\hat{D}$ $= \frac{1}{2}(7,5)(9,4)\sin 32°$ $= 18,7m^2$ OR Area of ΔDEC $= \frac{1}{2}CE.DC\sin 52,6°$ $= \frac{1}{2}(5,0)(9,4)\sin 52,6°$ $= 18,7m^2$	✓ substitution ✓answer (2)

11.4	$\sin 32° = \frac{EG}{7,5}$ $EG = 7,5.\sin 32°$ $= 4,0$ $EF = (4 + 3,5)$ $= 7,5$ metres **OR** $EG = EC.\sin 52,6°$ $= (5,0).\sin 52,6°$ $= 4,0$ $EF = 4,0 + 3,5$ $= 7,5$ **OR**	✓ratio ✓substitution ✓answer (3) **[11]**

	$\frac{1}{2}.DC.EG = \text{area } \Delta DEC$ $\frac{1}{2}(9,4)EG = 18,7$ $\therefore \ EG = \frac{18,7 \times 2}{9,4}$ $= 4,0$ $EF = 4,0 + 3,5$ $= 7,5$	

QUESTION 12

12.1	Period = 360°	✓answer (1)
12.2	Amplitude = $\frac{1}{2}$	✓✓answer (2)
12.3		✓shape ✓x intercepts ✓ amplitude (3)
12.4	2 solutions	✓answer (1)
12.5	$-60° \le x \le 120°$ or $x \in [-60°; 120°]$	✓ $-60°; 120°$ ✓ notation (2)
12.6	$-90° < x < 30°$ or $x \in (-90°; 30°)$	✓✓ $-90°; 30°$ ✓ notation (3) **[12]**

TOTAL: 150